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Simplified Approach of Jet Aerodynamics with a View to Acoustics

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Nomenclature

C_p = specific heat, J/kg · K or cal/g
c = speed of sound, m/s
D = jet diameter, m
H = total enthalpy, J/kg
L = distance from the nozzle
L_C = laminar core length
L_M = turbulence peak location
L_P = sound power peak location
L_S = jet supersonic length
M = Mach number
m = mass flow rate, kg/s
P = pressure, Pa
r = gas constant, J/kg · K
T = temperature, K
U = reduced velocity
V = velocity, m/s
α = mass fraction
γ = specific heat ratio
ξ = reduced abscissa

Subscripts

a = atmospheric (or pulled air) data
e = nozzle exit data
g = exhausted gas data
i = chamber stagnation conditions
j = fully expanded jet data
m = data at any point on the jet axis
t = nozzle throat data

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I. Introduction

A common engineering problem is to predict the jet noise of a given rocket engine, for instance to assess the vibroacoustic stress on a launcher at liftoff. For this purpose, semiempirical jet noise models are still preferred tools. NASA's model [1] of the 1970s has been used by the ONERA (France) for ARIANE launchers. In fact, several empirical improvements of this model have been necessary to reproduce the near sound field of static solid-propellant rocket engines or of experimental supersonic hot jets [2]. The apparent reason for the gaps of the initial model was an approach of jet aerodynamics which was too simplified. In Ref. [3] it is pointed out that several formulas quoted in the literature give contradictory results for the laminar core length, an often used aerodynamics reference length in jet aeroacoustics. Besides, the jet temperature effects are ignored or roughly estimated.

A small number of aerodynamics studies focused on the measurement or the estimate of the flow supersonic length [4,5]. In fact, this length is rarely taken into account in jet noise models [6]. Even so, it is admitted today that the supersonically convected eddies constitute a major noise source. Let us note that the supersonic length in acoustics may be defined according to the internal sound speed or to the ambient sound speed [3]. Several studies have been carried out in the U.S.A. [7] and in Russia [8] to determine more precisely the jet aerodynamics from experimental data. The jet aerodynamics model worked out at the TSNIIMASH (Russia) is detailed in Ref. [9]. Here we describe the simplified one-dimensional jet model developed at the ONERA on the basis of the latter model. In a first step, these models allow one to calculate the laminar core length and the noise peak position knowing the chamber conditions. It is generally admitted [1,9] that

$$L_P \approx 1.5 L_C \tag{1}$$

The knowledge of the noise peak location is of great interest: 1) It enables an accurate simulation of the near sound field because the noise peak is the origin of the main directivity lobe; and 2) it allows one to validate the models from some experimental studies [10,11]. The centerline velocity and temperature decays given by the models can also be compared with experimental data [12,13] and with results of CFD computations.

II. Simplified Jet Aerodynamics Model

The concept of ideal rocket [14] allows one to calculate the jet parameters for a given expansion ratio, knowing the chamber conditions and using the perfect-gas formalism. Thus, the fully expanded jet characteristics are calculated for an ideal jet having the same parameters (P_a, T_j, γ_j, r_j, and V_j) in a given cross section, the diameter of which is

$$D_j = 2\sqrt{\frac{m_t r_j T_j}{\pi P_a V_j}} \tag{2}$$

The idea is to generalize this approach by considering a fully expanded mixed jet where the parameters (P_a, T_m, γ_m, r_m, and V_m) for a given point of the jet axis remain constant on the entire cross section. By convenience, the gaseous products at the exhaust are called "gas" in the continuation. At any cross section of this ideal jet, we consider a mass fraction α of gas and a mass fraction $1-\alpha$ of mixed ambient air. Thus we have the mass relation

$$\alpha + (1 - \alpha) = 1 \tag{3}$$

knowing that $\alpha = 1$ in the laminar core, $\alpha = 0$ to infinity. Considering the conservation of momentum and of enthalpy, we can write

$$\alpha = V_m / V_j \tag{4}$$

$$\alpha H_g + (1 - \alpha) H_a = H_m \tag{5}$$

where V_m is the local velocity and H_x the total enthalpy (gas $x = g$, ambient air $x = a$, and air–gas mixture $x = m$), that is

$$H_x = Cp_x T_x + V_x^2 / 2 \tag{6}$$

According to Eq. (5), the local constant r_m and the local specific heat Cp_m of the mixture are given by (here the indices g and j are equivalent)

$$\alpha r_g + (1 - \alpha) r_a = r_m \tag{7}$$

$$\alpha Cp_g + (1 - \alpha) Cp_a = Cp_m \tag{8}$$

To introduce the local Mach number M_m, another equation taken from $M = V/c$ is used

$$V_m^2 = M_m^2 \gamma_m T_m \tag{9}$$

In this model, the specific heat ratio γ_m has a dynamic value dependent on the air–gas mixture ratio and also on the local temperature T_m, in accordance with the Le Châtelier and Mallard law (real-gas frozen flow calculation).

In theory the system of Eqs. (3–9) leads up to an equation of sixth degree in T_m but can be solved by iteration for a chosen Mach number M_m. Thus, every local parameter may be calculated, except the corresponding position L_m on the jet axis. For this purpose, the axial velocity decay of the Russian aerodynamics model is used (see Fig. 1). The position L_m is specified through its reduced abscissa

$$\xi = L_m / L_M \tag{10}$$

where L_M is the position of the turbulence peak likened to the noise peak L_P and given by the semiempirical equation

$$L_M / D_j = 6 + (8V_j^2/3 + 3Cp_a T_a)/(Cp_j T_j) \tag{11}$$

This equation is deduced from the Russian model and related to the fully expanded jet characteristics. The value of ξ depends on the reduced velocity U_m via the equations

$$U_m = V_m / V_j \tag{12}$$

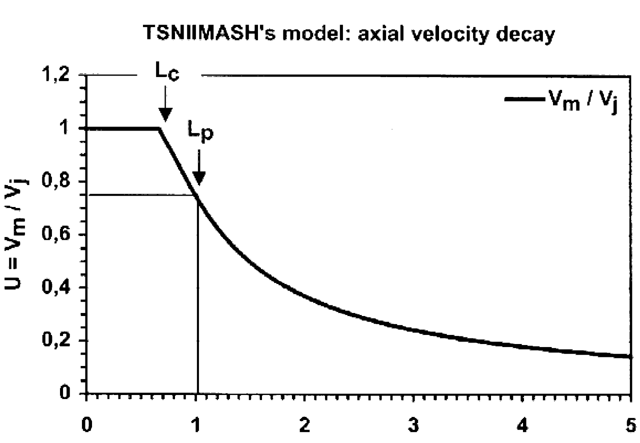

Fig. 1 Model of axial velocity decay developed at the TSNIIMASH.

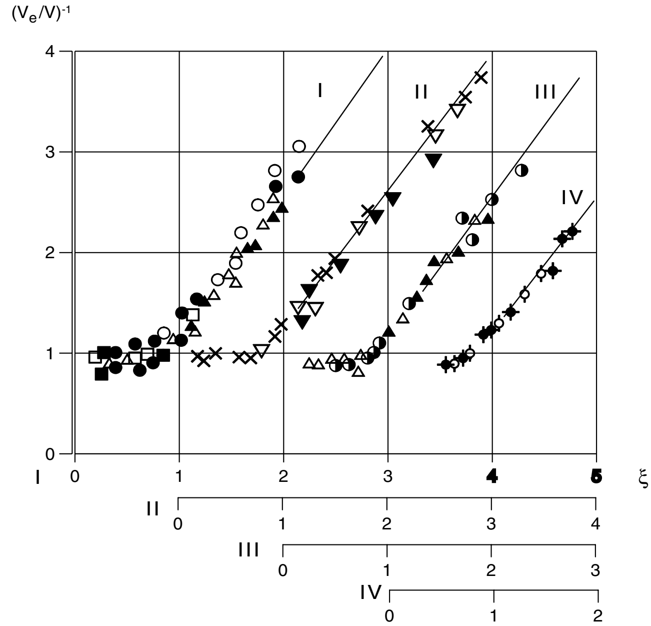

Fig. 2 Axial velocity decay for four series of jets as a function of ξ [8].

$$0 \le \xi \le \tfrac{2}{3} \quad \text{if } U_m = 1 \tag{13}$$

$$\xi = -\tfrac{4}{3} U_m + 2 \quad \text{if } \tfrac{3}{4} \le U_m < 1 \tag{14}$$

$$\xi = \frac{1}{2} - \frac{\log 2}{\log(1 - U_m)} \quad \text{if } U_m < \frac{3}{4} \tag{15}$$

Let us note that $\xi = 1$ corresponds to the turbulence or noise peak, where $U_M = 0.75$ (see Fig. 1). The empirical equations above come from numerous tests carried out in Russia with several types of jets (see Fig. 2).

An interesting possibility of the algebraic system is the calculation of the jet supersonic length by choosing either $M_m = 1$ or $V_m = c_a$ at the supersonic tip.

III. Experiments and Model Data

The Russian model of axial velocity decay can be validated thanks to well-known aerodynamics experiments with cold [12] or heated [13] supersonic air jets. The agreement between calculation by both Russian and French models and experimental data shown in Figs. 3 and 4 is good, particularly in the cold jet case. In Fig. 4 results obtained with CFD computer codes of ONERA [RANS k-ε, averaged large eddies simulation (LES)] are also indicated. For this hot jet ($T_i = 1370$ K), there is no available experimental data concerning the axial temperature. In Fig. 5, the temperature is expressed by convenience as a function of axial velocity. The agreement between the semiempirical models and the CFD computations appears satisfactory.

In Figs. 6 and 7, the experimental data concern a supersonic jet tested in the MARTEL test stand [2] of Centre National d'Etudes Spatiales. This jet, stemming from an air–hydrogen combustion, has a fully expanded Mach number of 2.8. Velocity and static temperature have been inferred from stagnation pressure and temperature. Several series of measurements have been done, which gives an idea of experimental and calculation uncertainties. In Fig. 6, we can see that Russian and French models give similar results close to experiment for the axial velocity. In Fig. 7, experimental results are very close to those of a CFD Russian computation (dashed line). The TSNIIMASH's model, based on a more complex aerodynamics formalism, gives good results far from the nozzle; the one-dimensional jet model of ONERA gives better results in the first part of the jet (another difference between the models is that the Russian

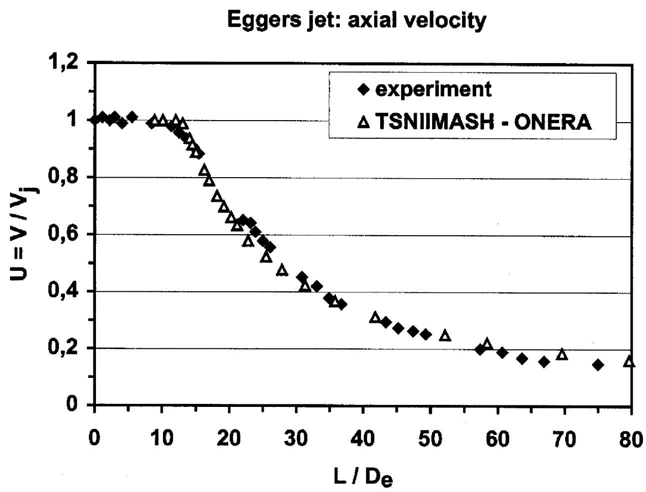

Fig. 3 Experimental and computed axial velocity decays for a Mach 2.2 cold jet.

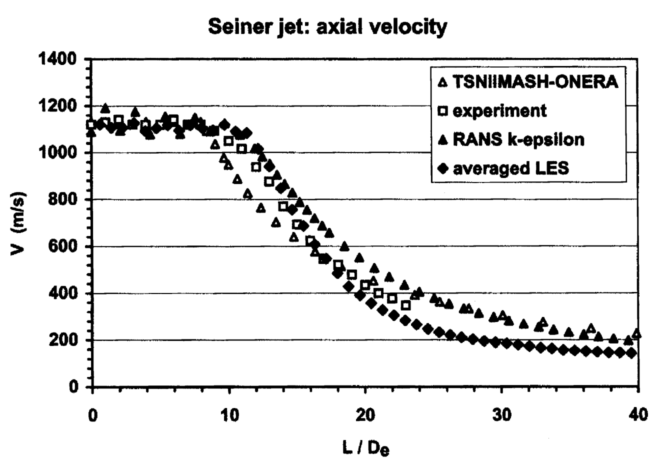

Fig. 4 Experimental and computed axial velocity decays for a Mach 2 hot jet.

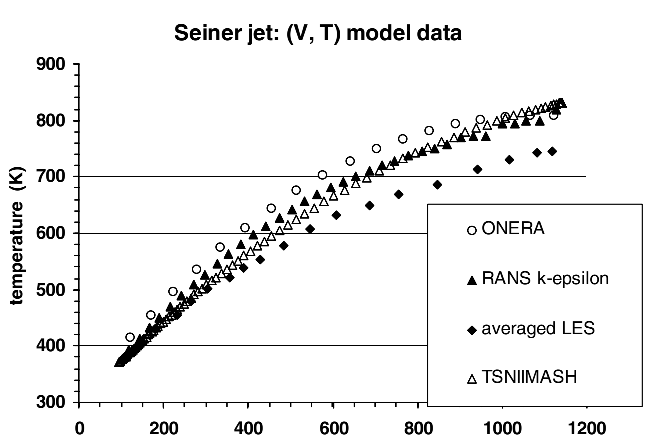

Fig. 5 Mach 2 hot jet computation: static temperature as a function of axial velocity.

model considers a mixture of perfect gases). Finally, we can conclude that the semiempirical model data are in accordance with both the experiment and CFD data.

Another validation of the models may be made from the estimates of the aerodynamics reference lengths found in the literature (laminar core length, supersonic length, and noise peak-to-nozzle distance).

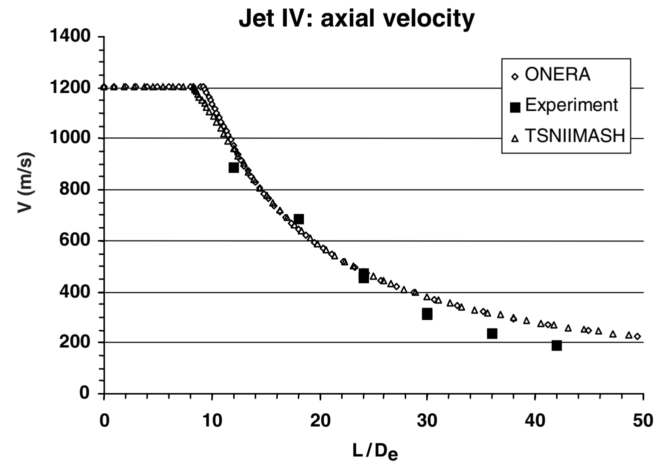

Fig. 6 Experimental and computed axial velocity decays for a Mach 2.8 hot jet.

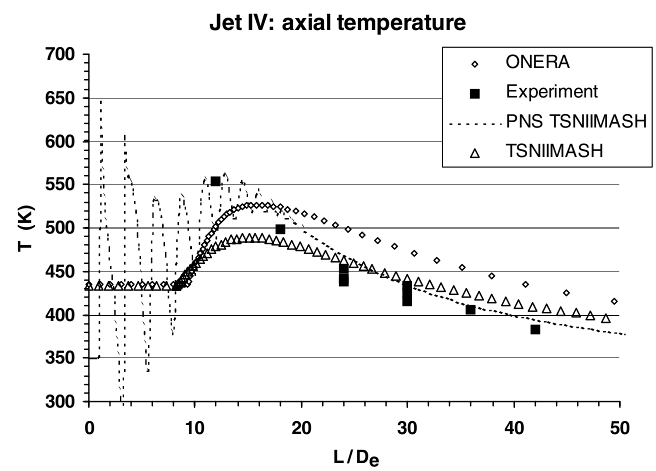

Fig. 7 Experimental and computed static temperature decays for a Mach 2.8 hot jet.

The measurements or estimates of these lengths are not so numerous [4–6]. Table 1 gives some references of cold jets for which experimental data are available. All jets are fully expanded at the nozzle exit and have relatively low Reynolds numbers ($3 \times 10^6 < Re < 3 \times 10^7$).

In Fig. 8 we have given, as a function of the Mach number, only the jet aerodynamics data (i.e., laminar core tip L_C, supersonic cone tip L_S, and turbulence peak $L_P = L_M$), which are clearly specified in the quoted papers (symbols in black). The corresponding computed data (symbols in white) are obtained with the help of both Russian and French models. Both models give quasi identical results for cold jets. The supersonic length is obtained by assuming that the local Mach number M_m is equal to 1 at the supersonic tip. The accordance between computed and experimental data is very satisfactory (standard deviation of differences is 13%). Note the accurate calculation of the sound power peak location for the Potter nitrogen jet ($M_j \approx 2.5$), determined with precision in a reverberant room [10].

Experimental data concerning hot jets or rocket jets are more difficult to find. Jets given in Table 2 have chamber temperatures between 1000 and around 2500 K. Several experimental jets have been tested at the ONERA in anechoic or semianechoic test facilities. Some aerothermodynamic data of military solid-propellant rocket engines have been found in Ref. [4,5,15].

As in Fig. 8, in Fig. 9 we show only the computed results that correspond to the experimental results available (symbols in black). According to experimental data, the computed data are referred to the nozzle exit diameter D_e but were in a first step calculated using the jet theoretical diameter D_j. Results stemming from Russian and French models are represented by identical

Table 1 Cold jets

Reference	Gas	M_j
Laufer et al. [11]	Air	1.47
Laufer et al. [11]	Air	1.97
Seiner et al. [13]	Air	2.00
Eggers [12]	Air	2.22
Laufer et al. [11]	Air	2.47
Potter [10]	Nitrogen	2.49

Table 2 Hot jets

Reference	Propellant	M_j
Varnier [2]	Propane–air	1.47
Seiner et al. [13]	Heated air	2.01
Varnier [2]	Hydrogen–air	2.83
Anderson and Johns [4]	Double-base	3.0
Mayes et al. [15]	Double-base	3.1
Anderson and Johns [4]	Double-base	3.35

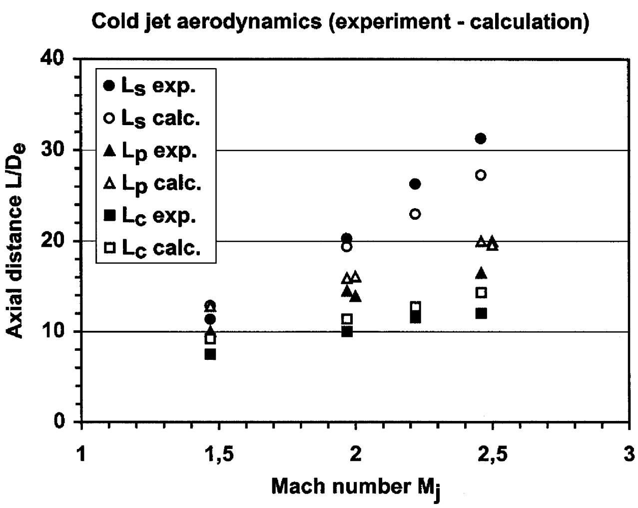

Fig. 8 Cold jets: measured (exp.) and computed (calc.) locations of the aerodynamics reference points.

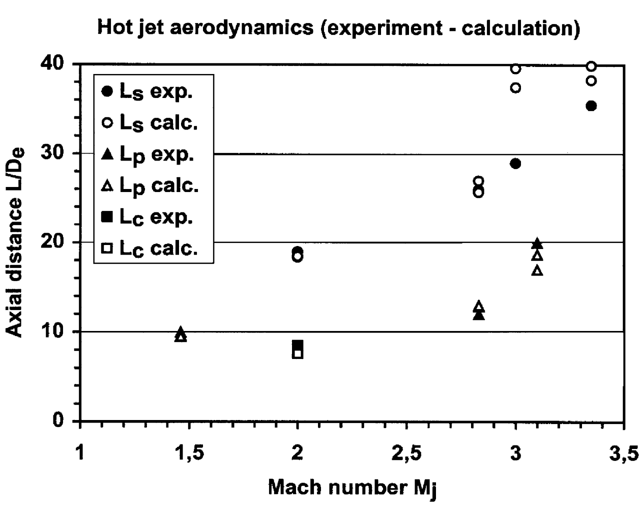

Fig. 9 Hot jets: measured (exp.) and computed (calc.) locations of the aerodynamics reference points.

symbols in white. We can conclude that both models give similar results for hot jets and that these results appear satisfactory in general.

Thus, the simplifications carried out in the ONERA's model do not modify the results very much on the one hand, and the improvements carried out (flow calculation with real gases) do not seem to give a great improvement on the other hand.

IV. Conclusion

The former semiempirical acoustic models for supersonic jets, stemmed from the subsonic model and based on experimental data having some disparity, were roughly related to jet aerodynamics. The recent computational aeroacoustics, which represents an opposite approach, utilizes complex theories and mathematical models that are not well adapted for a common engineering use.

In the present study, we propose an intermediate approach of a semiempirical type, which enables a quick and relatively precise calculation of all aerothermodynamic data along the jet axis, knowing the chamber characteristics and the composition of exhausted gases. More particularly, noise peak location, laminar core length, and jet supersonic length can be calculated accurately, which was not the case in former jet noise models and therefore constitutes the main contribution of the present paper.

Yet it remains to develop a complete jet noise model based on the local aerodynamics data calculated along the jet axis, and not only on the jet exhaust or fully expanded jet characteristics as is the case for all semiempirical models found in the literature.

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